# Revealing Complexity
Extended Abstract

Duarte Gonçalves[*] Salvatore Nunnari[†] Julen Zarate-Pina[‡]

It is consensual that complex decision problems lead to more mistakes. This phenomenon is well-documented in controlled experiments and real-world environments with high financial stakes, such as selecting health insurance or managing mortgages.[1] This has important implications, as misunderstanding complex incentive structures can lead to inconsistent choices and ultimately undermine the ability to infer preferences, measure expectations, predict behavior, forecast outcomes, and design effective policies and pricing schemes.

A fundamental first step is to assess if and when a decision problem is more complex than another. Researchers have made advances mostly conjecturing dimensions along which a problem in a specific domain becomes harder, corroborating these conjectures with objective measures of choice accuracy or effort. Difficulties arise exactly when choices are guided by the decision-maker's subjective preferences, making it difficult to tell whether choices are 'mistakes' or not — as happens in many cases of interest. Two approaches are then used, both presenting specific challenges. One approach extrapolates from *adjusted* similar problems in which choice accuracy or quality is measurable (e.g. Oprea, 2023; Enke et al., 2023; Enke and Shubatt, 2023). This methodology effectively requires changing the incentives and the nature of the problem to have straightforward 'right' and 'wrong' answers. The validity of the approach relies on assuming that it is relatively as easy to choose among alternatives in the original problem as it is to identify the correct alternative in the adjusted problem, which may not always be true.[2] A second approach — used in economics, psychology, and cognitive science — is to use typical measures

[*] Department of Economics, University College London; duarte.goncalves@ucl.ac.uk.
[†] Department of Economics, Bocconi University salvatore.nunnari@unibocconi.it.
[‡] Department of Economic Analysis, University of the Basque Country; julen.zarate-pina@ehu.eus.
*This draft*: 14 May 2024.

[1] For experimental evidence, see Rabin and Weizsäcker (2009), Martínez-Marquina et al. (2019), Enke (2020), Jin et al. (2022), Banovetz and Oprea (2023), Esteban-Casanelles and Gonçalves (2022), and Frydman and Nunnari (2023). For observational evidence from high stakes environments, see Sinaiko and Hirth (2011) and Bhargava et al. (2017) on health insurance, Choi et al. (2011) on pension plans, and Keys et al. (2016) and Agarwal et al. (2016) on mortgages.

[2] For instance, the complexity of choice under risk has been studied by asking which money lottery has the highest expected value — a question which has a correct answer, known to the analyst. At the same time, a decision-maker may be very unsure about whether lottery A has a higher expected value than B, while being certain that they prefer A to B.

of effort, such as decision time, with the underlying assumption that more complex problems require greater cognitive effort, resulting in slower response times (e.g. Forstmann et al., 2016; Caplin et al., 2020). However, recent literature suggests a more nuanced relationship between decision time and problem complexity, with fast decisions indicating very simple or very complex choices (Gonçalves, 2024).

This paper presents a novel methodology inspired by the incentive manipulation approach outlined in Gonçalves (2024), which leverages small changes in relative incentives to discern the complexity of decision problems. Building upon the sequential sampling framework, our methodology demonstrates that minor changes to incentives exert a disproportionately larger impact on choices in complex problems than in simpler ones. We validate this methodology using decision problems that we draw from established work in economics and psychology, and that can be unequivocally ranked in terms of complexity. We then apply our novel method to investigate how different features of the decision-making environment contribute to the complexity of choice across various domains of interest — including choice under certainty, choice under uncertainty, belief updating, and the choice of information sources — where introspection or previous work does not give us a definitive answer on what makes a problem complex. We also document the relationship between complexity and decision time and compare the complexity that can be inferred from choice problems with the complexity that can be inferred from adjusted problems with an objectively correct answer.

Our methodology relies on changing relative incentives to gauge problem complexity. We build on the groundwork in Gonçalves (2024), who considers a standard sequential sampling problem and investigates theoretically how problem complexity affects choice quality and decision time: choice quality decreases monotonically with complexity; decision time, on the other hand, is first increasing and then decreasing. This theoretical framework shows that response times are not a good measure of problem complexity and suggests an alternative to using choice accuracy (if unavailable): measuring how much choices respond to changes in incentives. Specifically, when offering decision-makers a small bonus for choosing a specific alternative leads them to choose this alternative more often (and faster — see Gonçalves 2023), then this small change to relative incentives is theoretically predicted to have a greater effect on choice in more complex problems than in simpler ones. Indeed, in simple problems, it is easy to identify the best available alternative, so a small (e.g., \$0.01) change to incentives will not make much difference.

We first provide an experimental validation of this new methodology. For this, we ask participants in our experiments to solve tasks that (a) have a unique correct answer and (b) we can unambiguously rank according to complexity. The complexity ranking we apply to these tasks is intuitive and relies on ample evidence from the psychology and economics literature.

The first problem is a classic numerosity discrimination task from the cognitive science literature. In this task, participants see a cloud of dots of two colors and guess which color is more frequent in this cloud. This task has been extensively used in economics to examine costly information acquisition (Dewan and Neligh, 2020; Dean and Neligh, 2023), and relies on the participants' perceptual capability to infer quantities. It has long been established that a smaller ratio between the number of balls of each color makes the problem more complex (Forstmann et al., 2016; Rodriguez and Ferreira, 2023), and there is also evidence that a larger overall number of dots influences the complexity of the problem (Clayton and Gilmore, 2015). We independently vary these two features of the task to create a natural ranking of complexity in the numerosity discrimination task: a smaller ratio and a larger number of dots induce higher complexity.

Second, we consider a task that requires simple arithmetic capabilities to be solved. The mental processing required to solve this task is more formal and less perceptual than the previous task, and, therefore, it allows us to investigate a different domain of cognitive processes. In this task, participants see simple arithmetic problems consisting of sums and subtractions and choose the problem with the highest value—similar to Caplin et al. (2011) and Jin et al. (2022). We vary the numerosity of mathematical operations and the number of alternatives participants consider to create versions of the task that we can rank in terms of complexity.

Third, we study the validity of our methodology in other problems that focus on computational mental processes, which have a natural complexity ordering. For this, we rely on the shell game, popular in fairs, in which a ball is placed under one of three cups, and, following a number of pairwise swaps of the cups, the participant is asked to guess under which cup it is. Complexity increases as the length of the number of steps increases (Stankov and Schweizer, 2007). In short, we use simple abstract tasks with a natural ranking of complexity to validate our hypotheses relating reaction to small incentives to complexity.

After validating our hypotheses with the abstract tasks described above, we categorize several economic problems in terms of their complexity using our methodology. The first domain we focus on is choice under risk, particularly choice between lotteries. We consider several variations and manipulations to assess which essential features make a lottery simpler or more complex — including support size, variability, similarity, compounding, dominance, and correlation — and contrast our findings with the existing literature.[3] We also examine how the framing of a lottery

[3]There is a large literature studying what makes choice over lotteries complex, such as lottery support size (Huck and Weizsäcker, 1999; Sonsino et al., 2002; Mononen, 2021; Fudenberg and Puri, 2022; Hu, 2023), whether lotteries are simple or compound (Wilcox, 1993; Kovářík et al., 2016), and the dissimilarity of available lotteries (Enke and

affects choice complexity, as in Tversky and Kahneman (1986) and Esponda and Vespa (2023).

We then turn to examining what makes information more complex. For that, we study both updating and the choice of information structure, as well as their association. We assess how complex a particular signal is for updating beliefs by modifying a belief elicitation task and varying the information structure signals are drawn. We specifically compare tasks in which we vary (1) whether signals are provided sequentially or all at once—re-examining the assumption that updating sequentially is more costly (see e.g. Bloedel and Zhong, 2021; Wong, 2024); (2) the number of different signal realizations (holding fixed the distribution over posterior beliefs); (3) whether the information structure provides direct or indirect information (see Gonçalves et al., 2023); and (4) the extent to which information entails nonlinear updating (Agranov and Reshidi, 2023). We then examine the choice of information structures in specific decision problems, contrasting these same features.

Finally, we use our methodology to analyze which elements of financial products, such as mortgages or pension funds, make choosing the optimal plan more complex. These products have multiple attributes that are complicated to aggregate. We vary the number and characteristics of attributes to generate a ranking of these attributes in terms of the complexity they generate.

Our preliminary results validate our hypotheses and show the promise of our approach. Specifically, we conducted pilot studies for two validation tasks discussed above, the numerosity discrimination task and the arithmetic task. We find that choice accuracy decreases in problem complexity in all our complexity variations. Moreover, in line with our hypotheses, response time is not monotone in problem complexity but rather hump-shaped: participants spend less time solving easy and complex problems and spend the most time in problems of intermediate complexity. Finally and most importantly, we observe evidence of greater sensitivity to incentives in more complex tasks.

In this project, we propose an innovative experimental methodology to measure a decision problem's complexity and to gauge how features of the decision-making environment contribute to it. We achieve this goal by proposing a behavioral marker theoretically associated with greater complexity: the sensitivity to small economic incentives. We first validate our methodology by documenting the sensitivity to small economic incentives in tasks whose complexity is unequivocal and established in the literature. We then provide evidence on economic problems whose complexity is ex-ante ambiguous by analyzing the sensitivity to small economic incentives in different judgment and decision-making environments (choice over lotteries, choice over infor-

Shubatt, 2023).

mation structures, and choice over financial products).

## References


**Agarwal, S., R. J. Rosen, and V. Yao.** 2016. "Why Do Borrowers Make Mortgage Refinancing Mistakes?" *Management Science* 62 (12): 3494–3509. 10.1287/mnsc.2015.2272.

**Agranov, M., and P. Reshidi.** 2023. "Disentangling Suboptimal Updating: Complexity, Structure, and Sequencing." *Working Paper.*

**Banovetz, J., and R. Oprea.** 2023. "Complexity and Procedural Choice." *American Economic Journal: Microeconomics* 15 (2): 384–413. 10.1257/mic.20210032.

**Bhargava, S., G. Loewenstein, and J. Sydnor.** 2017. "Choose to Lose: Health Plan Choices from a Menu with Dominated Option." *Quarterly Journal of Economics* 132 (3): 1319–1372. 10.1093/qje/qjx011.

**Bloedel, A., and W. Zhong.** 2021. "The Cost of Optimally Acquired Information." *Working Paper* 1–105.

**Caplin, A., D. Csaba, J. Leahy, and O. Nov.** 2020. "Rational Inattention, Competitive Supply, and Psychometrics." *Quarterly Journal of Economics* 135 (3): 1681–1724. 10.1093/qje/qjaa011.

**Caplin, A., M. Dean, and D. Martin.** 2011. "Search and Satisficing." *American Economic Review* 101 (7): 2899–2922. 10.1257/aer.101.7.2899.

**Choi, J. J., D. Laibson, and B. C. Madrian.** 2011. "$100 Bills on the Sidewalk: Suboptimal Investment in 401(k) Plans." *Review of Economics and Statistics* 93 (3): 748–763. 10.1162/REST_a_00100.

**Clayton, S., and C. Gilmore.** 2015. "Inhibition in dot comparison tasks." *ZDM Mathematics Education* 47 759–770. 10.1007/s11858-014-0655-2.

**Dean, M., and N. Neligh.** 2023. "Experimental Tests of Rational Inattention." *Journal of Political Economy* 131 (12): 3415–3461. 10.1086/725174.

**Dewan, A., and N. Neligh.** 2020. "Estimating information cost functions in models of rational inattention." *Journal of Economic Theory* 187 (105011): 1–32. 10.1016/jet.2020.105011.

**Enke, B.** 2020. "What You See Is All There Is." *Quarterly Journal of Economics* 135 (3): 1363–1398. 10.1093/qje/qjaa012.

**Enke, B., T. Graeber, and R. Oprea.** 2023. "Complexity and Time." *NBER Working Paper* 31047 1–64. 10.3386/w31047.

**Enke, B., and C. Shubatt.** 2023. "Quantifying Lottery Choice Complexity." *NBER Working Paper* 31677 1–63. 10.3386/w31677.

**Esponda, I., and E. Vespa.** 2023. "Contingent Thinking and the Sure-Thing Principle: Revisiting

Classic Anomalies in the Laboratory." *The Review of Economic Studies* rdad102. 10.1093/restud/rdad102.

**Esteban-Casanelles, T., and D. Gonçalves.** 2022. "The Effect of Incentives on Choices and Beliefs in Games: An Experiment." *Working Paper.*

**Forstmann, B., R. Ratcliff, and E.-J. Wagenmakers.** 2016. "Sequential Sampling Models in Cognitive Neuroscience: Advantages, Applications, and Extensions." *Annual Review of Psychology* 67 (1): 641–666. 10.1146/annurev-psych-122414-033645.

**Frydman, C., and S. Nunnari.** 2023. "Coordination with Cognitive Noise." *Working Paper.*

**Fudenberg, D., and I. Puri.** 2022. "Simplicity and Probability Weighting in Choice Under Risk." *American Economic Review: Papers and Proceedings* 122 421–425.

**Gonçalves, D.** 2023. "Sequential Sampling Equilibrium." *Working Paper* 1–57. 10.48550/arXiv.2212.07725.

**Gonçalves, D.** 2024. "Speed, Accuracy, and Complexity." 1–32. 10.48550/arXiv.2403.11240.

**Gonçalves, D., J. Libgober, and J. Willis.** 2023. "Retractions: Learning from Information about Information." *Working Paper* 1–57. 10.48550/arXiv.2106.11433.

**Hu, E. H.** 2023. "A Procedural Model of Complexity Under Risk." *Working Paper.*

**Huck, S., and G. Weizsäcker.** 1999. "Risk, complexity, and deviations from expected-value maximization: Results of a lottery choice experiment." *Journal of Economic Psychology* 20 699–715.

**Jin, G. Z., M. Luca, and D. Martin.** 2022. "Complex Disclosure." *Management Science* 68 (5): 3175–3973. 10.1287/mnsc.2021.4037.

**Keys, B. J., D. G. Pope, and J. C. Pope.** 2016. "Failure to Refinance." *Journal of Financial Economics* 122 (3): 482–499. 10.1016/j.jfineco.2016.01.031.

**Kovářík, J., D. Levin, and T. Wang.** 2016. "Ellsberg paradox: Ambiguity and complexity aversions compared." *Journal of Risk and Uncertainty* 52 47–64. 10.1007/s11166-016-9232-0.

**Martínez-Marquina, A., M. Niederle, and E. Vespa.** 2019. "Failures in Contingent Reasoning: The Role of Uncertainty." *American Economic Review* 109 (10): 3437–3474. 10.1257/aer.20171764.

**Mononen, L.** 2021. "On preference for simplicity and probability weighting." *Working Paper.*

**Oprea, R.** 2023. "Simplicity Equivalents." *Working Paper* 1–64.

**Rabin, M., and G. Weizsäcker.** 2009. "Narrow Bracketing and Dominated Choices." *American Economic Review* 99 (4): 1508–1534. 10.1257/aer.99.4.1508.

**Rodriguez, C., and R. Ferreira.** 2023. "To what extent is dot comparison an appropriate measure of approximate number system?" *Frontiers in Psychology* 13 1–10. 10.3389/fpsyg.2022.1065600.

**Sinaiko, A. D., and R. A. Hirth.** 2011. "Consumers, health insurance and dominated choices." *Journal of Health Economics* 30 (2): 450–457. 10.1016/j.jhealeco.2010.12.008.

**Sonsino, D., U. Benzion, and G. Mador.** 2002. "The Complexity Effects on Choice with Uncertainty – Experimental Evidence." *The Economic Journal* 112 (482): 936–965. 10.1111/1468-0297.00073.

**Stankov, L., and K. Schweizer.** 2007. "Raven's progressive matrices, manipulations of complexity and measures of accuracy, speed and confidence." *Experimental Economics* 49-4 326––342.

**Tversky, A., and D. Kahneman.** 1986. "Rational Choice and the Framing of Decisions." *The Journal of Business* 59 (4): S251–S278.

**Wilcox, N.** 1993. "Lottery choice: Incentives complexity and decision time." *Economic Journal* 103 (421): 1397–1417. 10.2307/2234473.

**Wong, Y. F.** 2024. "Dynamic Monitoring Design." *Working Paper* 1–64.